FORMULATION OF CORRECTOR METHODS FROM 3- STEP HYBRID ADAMS TYPE METHODS FOR THE SOLUTION OF FIRST ORDER ORDINARY DIFFERENTIAL EQUATION

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Abstract- This paper focuses on the formulation of 3-step hybrid Adams type method for the solution of first order differential equation (ODE). The methods which was derived on both grid and off grid points using multistep collocation schemes and also evaluated at some points to produced Block Adams type method and Adams moulton method respectively. The method with the highest order was selected to serve as the corrector. The convergence was valid and efficient. The numerical experiments were carried out and reveal that hybrid Adams type methods performed better than the conventional Adams moulton method.

Keywords- Adam-Moulton Type (AMT), Corrector Method, Off-grid, Block Method, Convergence Analysis.

I. INTRODUCTION

The methods of Euler, Heun, Taylor and Runge-Kutta are called single-step methods because they use only the information from one previous point to compute the successive point, that is, only the initial point (t_0, y_0) is used to compute (t_1, y_1) and in general y_k is needed to compute y_{k+1}. The idea of extending this method was developed by Bashforth and Adams in (1883) that is, approximating the solution at a point to depend on the solution values at several previous step values,while this was later developed by Moulton in 1926. There are two types of Adams methods, the explicit and the implicit types. The explicit type is called the Adams-Bashforthmethods and the implicit type is called the Adams-Moulton methods. The Adams Moulton method which is of the form:

$$y_{n+k} - y_{n+k-1} = h\sum_{j=0}^{k} \alpha_i f_{n+i} \qquad (1.0)$$

The methods are all zero stable since all the spurious roots of $\rho(\varepsilon)$ are located at the origin. These methods were widely used in the past for approximating the solutions of non–stiff ordinarily differential equations .Also many Researchers have worked extensively in this area such as Awoyemi [1], Yahayaand Sagir, Yahaya, Sokoto, and Shaba, Oluwale, Badmus and Mshelia[2],[3], Badmus and Adegboye [4],Badmus et al [5], Odekunle et al ([7],[8], Yahaya and Adegboye [10], to mention but a few.

This paper intends to derive and compare the Block Adams Method and a Block Adams Type Method all at k =3 from their continuous schemes respectively at both grid and off-grid points to obtain the new discrete schemes, it develop a high order, zero stable and consistent block method and use it to solve some existing known problems to ascertain the level of convergence.

Definition 1.0: One-Step Method

The construct an approximate solution$x_{k+1} = x(t)_k$,using only one previous approximation x_k. The approach in this method enjoys the virtue that the step size (h) can be changed at every iteration, if desired, thus providing a mechanism for error control. A general expression of one-step method is:

$$y_{n+1} = y_n + hf(x_n, y_n) \text{ where } f(x_n, y_n, h) = f_n = f(x_n, y_n) \text{(Lambert [6])}$$

Definition 1.2: Linear Multistep Method (LMM)

If a computational method for determining a sequence between $[y_n]$takes the form of a linear relationship betweeny_{n+j}, f_{n+j}, j = 0,1,2, …,k, then we call it a LMM of step number K or a linear k-step method.

A linear k-step method is mathematically defined as:

$$a_k y_{n+k} + a_{k-1} y_{n+k-1} \ldots\ldots + a_1 y_{n+1} + a_0 y_n = h(\beta_k f_{n+k} + \ldots\ldots + \beta_1 f_{n+1} + \beta_0 f_n)$$

Which can be written in compartment form as:

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j} = h \qquad 1.1$$

Where $|\alpha_k|$and $|\beta_k| \neq 0$ and $\beta_k = 1$when$\beta_k \neq 0$, the scheme becomes an implicit scheme, otherwise explicit scheme. Subair [9]

Definition 1.3: Convergence

The block corrector is convergent by the consequence of Dahlquist theorem given below.

Theorem:

The necessary and sufficient conditions that a continuous LMM be convergent are that it be consistent and zero-stable.

Definition 1.3: Zero Stability

The linear multistep method (1.2) is said to satisfy the root conditions if all the roots of the first characteristics polynomial have modulus less than or equal to unity and those of modulus unity are simple. The method (1.2) is said to be zero stable if it satisfies the root condition Lambert [6].

Definition 1.4: Consistent Lambert [8]

The linear multistep method (1.2) is said to be consistent if it has order p ≥ 1, that is, if

$$\sum_{j=0}^{k} \alpha_j = 0 \qquad 1.2$$

And;

$$\sum_{j-0}^{k} j\,\alpha_j - \sum_{j-0}^{k} \beta_j = 0 \qquad 1.3$$

Introducing the first and second characteristics polynomials (1.2), we have from (1.3) LMM type (1.2) is consistent if $\rho(1) = 0, \rho'(1) = \delta(1)$

II. METHODOLOGY

Given a power series of the form:

$$p(x) = \sum_{j=0}^{\infty} \alpha_j x^j$$

Which is used as our basis to produce an approximate solution to (1.0) as:

$$y(x) = \sum_{j=0}^{8+t-1} \alpha_j x^j \qquad (2.1)$$

and;

$$y'(x) = \sum_{j=0}^{8+t-1} j\alpha_j x^j = f(x,y) \qquad (2.2)$$

Where α_j'sare the parameters to be determined, and are the points of collocation and interpolation respectively. This process leads to (s + t – 1) non-linear system of equations with (s + t – 1) unknown coefficients, which are to be determined by the use of maple 17 mathematical software.

2.1 Three-Step Adams -Moulton Hybrid Type Methodwith Two Off-Grids

Using equations (2.1) and (2.2), s = 6, t = 1. Our choice of degree of polynomial is (s+t–1). Equations (2.1) and (2.2) are interpolated and collocated respectively at the points$x = \left(x_n, x_{n+\frac{3}{2}}, x_{n+2}, x_{n+\frac{5}{2}}, x_{n+3}\right)$which gives the following non-linear system of equations of the form:

$$\sum_{j=0}^{8+t-1} \alpha_j x^j = y_{n+i}$$

$$\sum_{j=1}^{8+t-1} j\alpha_j x^{j-1} = f_{n+i} \qquad \text{where i = (0,3/2,2,5/2,3)}$$

With the mathematical software, we obtain the continuous formulation of equations (2.3) and (2.4) as follows:

$$\begin{aligned}
y(x) = {} & f_n\Bigg(-\frac{1}{1080}\frac{(x_n+h)(297h^5+783h^4x_n+783h^3x_n^2+377h^2x_n^3+88hx_n^4+8x_n^5)}{h^5} \\
& +\frac{1}{90}\frac{(90h^5+261h^4x_n+290h^3x_n^2+155h^2x_n^3+40hx_n^4+4x_n^5)x}{h^5} \\
& -\frac{1}{180}\frac{(261h^4+580h^3x_n+465h^2x_n^2+160hx_n^3+20x_n^4)x^2}{h^5} \\
& +\frac{1}{54}\frac{(58h^3+93h^2x_n48hx_n^2+8x_n^3)x^3}{h^5}-\frac{1}{72}\frac{(31h^2+32hx_n+8x_n^2)x^4}{h^5} \\
& +\frac{2}{45}\frac{(x_n+2h)x^5}{h^5}-\frac{1}{135}\frac{x^6}{h^5}\Bigg) \\
& +f_{n+1}\Bigg(-\frac{1}{360}\frac{(x_n+h)(673h^5-673h^4x_n-2027h^3x_n^2-1393h^2x_n^3-392hx_n^4-40x_n^5)}{h^5} \\
& -\frac{1}{6}\frac{x_n(90h^4x_n+171h^3x_n+119h^2x_n^2+36hx_n^3+4x_n^4)x}{h^5} \\
& +\frac{1}{12}\frac{(90h^4+342h^3x_n+357h^2x_n^2+144hx_n^3+20x_n^4)x^2}{h^5} \\
& -\frac{1}{18}\frac{(171h^3+357h^2x_n+216hx_n^2+40x_n^3)x^3}{h^5} \\
& +\frac{1}{24}\frac{(119h^2+144hx_n+40x_n^2)x^4}{h^5}-\frac{2}{15}\frac{(5x_n+9h)x^5}{h^5}+\frac{1}{9}\frac{x^6}{h^5}\Bigg) \\
& +f_{n+2}\Bigg(-\frac{1}{120}\frac{(x_n+h)(211h^5-211h^4x_n-1139h^3x_n^2-1021h^2x_n^3-344hx_n^4-40x_n^5)}{h^5} \\
& -\frac{1}{2}\frac{x_n(45h^4+108h^3x_n+91h^2x_n^2+32hx_n^3+4x_n^4)x}{h^5} \\
& +\frac{1}{4}\frac{(45h^4+216h^3x_n+273h^2x_n^2+128hx_n^3+20x_n^4)x^2}{h^5} \\
& -\frac{1}{6}\frac{(108h^3+273h^2x_n+192hx_n^2+40x_n^3)x^3}{h^5}+\frac{1}{8}\frac{(91h^2+128hx_n+40x_n^2)x^4}{h^5} \\
& \frac{2}{5}\frac{(5x_n+8h)x^5}{h^5}\ \ \frac{1}{3}\frac{x^6}{h^5}\Bigg) \\
& +f_{n+3}\Bigg(-\frac{1}{1080}\frac{(x_n+h)(129h^5-129h^4x_n-771h^3x_n^2-769h^2x_n^3-396hx_n^4-40x_n^5)}{h^5} \\
& -\frac{1}{18}\frac{x_n(30h^4 77h^3x_n+71h^2x_n^2+28hx_n^3+4x_n^4)x}{h^5} \\
& +\frac{1}{36}\frac{(30h^4+154h^3x_n+213h^2x_n^2+112hx_n^3+20x_n^4)x^2}{h^5} \\
& -\frac{1}{54}\frac{(77h^3+213h^2x_n+168hx_n^2+40x_n^3)x^3}{h^5} \\
& +\frac{1}{72}\frac{(71h^2+112hx_n+40x_n^2)x^4}{h^5}-\frac{2}{5}\frac{(5x_n+7h)x^5}{h^5}+\frac{1}{27}\frac{x^6}{h^5}\Bigg) \\
& +f_{n+\frac{3}{2}}\Bigg(\frac{8}{9}\frac{(x_n+h)(39h^5-39h^4x_n-186h^3x_n^2-149h^2x_n^3-46hx_n^4-5x_n^5)}{h^5} \\
& +\frac{8}{9}\frac{x_n(30h^4x_n+67h^3x_n+52h^2x_n^2+17hx_n^3+2x_n^4)x}{h^5} \\
& -\frac{8}{9}\frac{(15h^4+67h^3x_n+78h^2x_n^2+34hx_n^3+5x_n^4)x^2}{h^5} \\
& +\frac{8}{27}\frac{(67h^3+156h^2x_n+102hx_n^2+20x_n^3)x^3}{h^5}-\frac{8}{9}\frac{(93h^2+17hx_n+5x_n^2)x^4}{h^5} \\
& +\frac{8}{45}\frac{(10x_n+17h)x^5}{h^5}-\frac{8}{27}\frac{x^6}{h^5}\Bigg) \\
& +f_{n+\frac{5}{2}}\Bigg(\frac{8}{45}\frac{(x_n+h)(4h^5-4h^4x_n-23h^3x_n^2-22h^2x_n^3-8hx_n^4-x_n^5)}{h^5} \\
& +\frac{8}{15}\frac{x_n(18x_n+45h^3x_n+40h^2x_n^2+15hx_n^3+2x_n^4)x}{h^5} \\
& -\frac{8}{15}\frac{(9h^4+45h^3x_n+60h^2x_n^2+30hx_n^3+5x_n^4)x^2}{h^5} \\
& +\frac{8}{9}\frac{(9h^3+24h^2x_n+18hx_n^2+4x_n^3)x^3}{h^5}-\frac{8}{3}\frac{(2h^2+3hx_n+x_n^2)x^4}{h^5} \\
& +\frac{8}{15}\frac{(2x_n+3h)x^5}{h^5}-\frac{8}{45}\frac{x^6}{h^5}\Bigg)+y_{n+1}
\end{aligned}$$

When equation (2.5) evaluated at $x = x_{n+j}$where$j = 0, \frac{3}{2}, 2, \frac{5}{2}, 3$ and its first derivative gives the following set of discrete schemes to form the first hybrid block method atk = 3.

$$y_n := -\frac{11}{40}hf_n - \frac{673}{360}hf_{n+1} + \frac{211}{120}hf_{n+2} - \frac{43}{360}hf_{n+3} + \frac{104}{45}hf_{n+\frac{3}{2}} + \frac{32}{45}hf_{n+\frac{5}{2}} + y_{n+1}$$

$$y_{n+\frac{3}{2}} := -\frac{1}{640}hf_n + \frac{1139}{5760}hf_{n+1} + \frac{217}{1920}hf_{n+2} - \frac{31}{5760}hf_{n+3} + \frac{139}{360}hf_{n+\frac{3}{2}} + \frac{13}{360}hf_{n+\frac{5}{2}} + y_{n+1}$$

$$y_{n+2} := -\frac{1}{1080}hf_n - \frac{7}{40}hf_{n+1} + \frac{7}{40}hf_{n+2} - \frac{1}{1080}hf_{n+3} + \frac{88}{135}hf_{n+\frac{3}{2}} + y_{n+1}$$

$$y_{n+\frac{5}{2}} = -\frac{1}{640}hf_n + \frac{123}{640}hf_{n+1} + \frac{333}{640}hf_{n+2} - \frac{7}{640}hf_{n+3} + \frac{23}{40}hf_{n+\frac{3}{2}} + \frac{9}{40}hf_{n+\frac{5}{2}} + y_{n+1}$$

$$y_{n+3} := -\frac{7}{45}hf_{n+1} - \frac{4}{15}hf_{n+2} + \frac{7}{45}hf_{n+3} - \frac{32}{45}hf_{n+\frac{3}{2}} + \frac{32}{45}hf_{n+\frac{5}{2}} + y_{n+1}$$

Equations (2.6) are of uniform order 7, with error constant as follows:

$$\left[\frac{697}{241920}, \frac{2951}{30965760}, \frac{1}{26880}, \frac{19}{163840}\right]$$

The Second Adam Block Scheme at $k=3$ derived as follows:

Equation (2.1) is interpolated at $x = x_{n+j}$, j =1 and equations (2.2) is collocate at $x = x_{n+j}$ for j=0,2,3 which gives the system of non-linear equations of the form 8

$$\sum_{j=0}^{s+t-1} \alpha^j x^j = y_{n+i} \qquad i = 0 \qquad (2.9)$$

$$\sum_{j=0}^{s+t-1} j\alpha^j x^{j-1} = f_{n+i} \qquad i = 0, 1, 2, 3 \qquad (3.0)$$

With the use of maple 17 mathematical software, we obtain the continuous formula for the method as:

$$\begin{aligned} y(x) = {} & f_n\Big(-\frac{1}{24}\frac{(x_n+h)(9h^3+15h^2x_n+7hx_n^2+x_n^3)}{h^3}+\frac{1}{6}\frac{(6h^3+11h^2x_n+6hx_n^2+x_n^3)x}{h^3} \\ & -\frac{1}{12}\frac{(11h^2+12hx_n+3x_n^2)x^2}{h^3}\Big)+\frac{1}{6}\frac{(x_n+2h)x^3}{h^3}-\frac{1}{24}\frac{x^4}{h^3} \\ & + f_{n+1}\Big(-\frac{1}{24}\frac{(x_n+h)(19h^3-19h^2x_n+17hx_n^2+3x_n^3)}{h^3}+\frac{1}{2}\frac{x_n(6h^2+5hx_n+x_n^2)x}{h^3} \\ & +\frac{1}{4}\frac{(6h^2+10hx_n+3x_n^2)x^2}{h^3}-\frac{1}{6}\frac{(5h+3x_n)x^3}{h^3}+\frac{1}{8}\frac{x^4}{h^3}\Big) \\ & + f_{n+2}\Big(\frac{1}{24}\frac{(x_n+h)(5h^3-5h^2x_n-13hx_n^2-3x_n^3)}{h^3}+\frac{1}{2}\frac{x_n(3h^2+4hx_n+x_n^2)x}{h^3} \\ & -\frac{1}{4}\frac{(3h^2+8hx_n+3x_n^2)x^2}{h^3}+\frac{1}{6}\frac{(4h+3x_n)x^3}{h^3}-\frac{1}{8}\frac{x^4}{h^3}\Big) \\ & + f_{n+3}\Big(-\frac{1}{24}\frac{(x_n+h)(h^3-h^2x_n-3hx_n^2-x_n^3)}{h^3}-\frac{1}{6}\frac{x_n(2h^2+3hx_n+x_n^2)x}{h^3} \\ & +\frac{1}{12}\frac{(2h^2+6hx_n+3x_n^2)x^2}{h^3}-\frac{1}{6}\frac{(x_n+h)x^3}{h^3}+\frac{1}{24}\frac{x^4}{h^3}\Big)+y\ldots \end{aligned}$$

When equation (2.5) evaluated at $\boldsymbol{x = x_{n+j}}$ where $\boldsymbol{j = 0, 2, 3}$ and its first derivative gives the following set of discrete schemes to form the hybrid block method at k = 3.

$$y_n := -\frac{3}{8}hf_n - \frac{19}{24}hf_{n+1} + \frac{5}{24}hf_{n+2} - \frac{1}{24}hf_{n+3} + y_{n+1}$$

$$y_{n+2} = -\frac{1}{24}hf_n + \frac{13}{24}hf_{n+1} + \frac{13}{24}hf_{n+2} - \frac{1}{24}hf_{n+3} + y_{n+1}$$

$$y_{n+3} = \frac{1}{3}hf_{n+1} - \frac{4}{3}hf_{n+2} + \frac{1}{3}hf_{n+3} + y_{n+1}$$

The proposed schemes in (3.5) are of order $[4, 4, 4]^T$ and error constant$[\frac{19}{720}, \frac{11}{720}, -\frac{1}{90}]$.

III. BLOCK ANALYSIS OF THE METHODS

$$\begin{pmatrix} -10000 \\ -11000 \\ -10100 \\ -10010 \\ -10001 \end{pmatrix} \begin{pmatrix} y_{n+1} \\ y_{n+3/2} \\ y_{n+2} \\ y_{n+\frac{5}{2}} \\ y_{n+3} \end{pmatrix} = \begin{pmatrix} 00001 \\ 00000 \\ 00000 \\ 00000 \\ 0\ 0\ 0\ 0\ 0 \end{pmatrix} \begin{pmatrix} y_{n-\frac{5}{2}} \\ y_{n-2} \\ y_{n-\frac{3}{2}} \\ y_{n-1} \\ y_n \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{-673}{360} & \frac{104}{45} & \frac{-211}{120} & \frac{32}{45} & \frac{-43}{360} \\ -\frac{1139}{5760} & \frac{139}{360} & -\frac{217}{1920} & \frac{13}{360} & \frac{-31}{5760} \\ \frac{7}{40} & \frac{139}{360} & \frac{7}{40} & 0 & \frac{-1}{1080} \\ \frac{123}{640} & \frac{23}{40} & \frac{333}{640} & \frac{9}{40} & -\frac{7}{640} \\ \frac{7}{45} & \frac{32}{45} & \frac{4}{15} & \frac{32}{45} & \frac{7}{45} \end{pmatrix} \begin{pmatrix} f_{n+1} \\ f_{n+3/2} \\ f_{n+2} \\ f_{n+\frac{5}{2}} \\ f_{n+3} \end{pmatrix} + \begin{pmatrix} 0000\,\frac{11}{40} \\ 0000\,\frac{35}{128} \\ 0000\,\frac{37}{135} \\ 0000\,\frac{35}{128} \\ 0\ 0\ 0\ 0\ \frac{11}{40} \end{pmatrix} \begin{pmatrix} f_{n-\frac{5}{2}} \\ f_{n-2} \\ f_{n-\frac{3}{2}} \\ f_{n-1} \\ f_n \end{pmatrix}$$

Let $A^{(0)} = \begin{pmatrix} -10000 \\ -11000 \\ -10100 \\ -10010 \\ -10001 \end{pmatrix} A^{(1)} = \begin{pmatrix} 00001 \\ 00000 \\ 00000 \\ 00000 \\ 0\ 0\ 0\ 0\ 0 \end{pmatrix}$

$$B^{(0)} = \begin{pmatrix} \frac{-673}{360} & \frac{104}{45} & \frac{-211}{120} & \frac{32}{45} & \frac{-43}{360} \\ -\frac{1139}{5760} & \frac{139}{360} & -\frac{217}{1920} & \frac{13}{360} & \frac{-31}{5760} \\ \frac{7}{40} & \frac{139}{360} & \frac{7}{40} & 0 & \frac{-1}{1080} \\ \frac{123}{640} & \frac{23}{40} & \frac{333}{640} & \frac{9}{40} & -\frac{7}{640} \\ \frac{7}{45} & \frac{32}{45} & \frac{4}{15} & \frac{32}{45} & \frac{7}{45} \end{pmatrix} B^{(1)} = \begin{pmatrix} 0000\,\frac{11}{40} \\ 0000\,\frac{35}{128} \\ 0000\,\frac{37}{135} \\ 0000\,\frac{35}{128} \\ 0\ 0\ 0\ 0\ \frac{11}{40} \end{pmatrix}$$

We shall normalize the block method (3.1) by multiplying matrices,$A^{(0)}, A^{(1)}, B^{(0)}, B^{(1)}$,, with inverse of $A^{(0)}$to obtain an$A'^{(0)}, A'^{(1)}, B'^{(0)}$ *and* $B'^{(1)}$, Then,P(R)=det $[A'^{(0)} - A'^{(1)}]$

$$= det[R \begin{pmatrix} 10001 \\ 01000 \\ 00100 \\ 00010 \\ 0\ 0\ 0\ 0\ 1 \end{pmatrix} - \begin{pmatrix} 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,1 \end{pmatrix} = 0$$

$$= det\left[\begin{pmatrix} R & 0 & 0\,0 & -1 \\ 0 & R & 0\,0 & -1 \\ 0 & 0 & R\,0 & -1 \\ 0 & 0 & 0\,R & -1 \\ 0 & 0 & 0\,0 & R-1 \end{pmatrix}\right] = R^5 - R^4 = 0$$

Which implies that R1 = R2 = R3 = R4 = 0 and R5 =1. Hence from the definition (1.3) equation, the method (3.1) is zero stable and also consistent as its order is5>1 ,thus convergent.

The same analysis holds for block methods (2.10) and (2.12), thus they are zero stable and convergent.

IV. NUMERICAL EXPERIMENTS

The two block methods derived at$k=3$, are demonstrated with the following problems:

Problem 1: $y^1 = 0.5(1-y), y(0) = 0.5, h = 0.1$

Exact solution:$y(x) = 1 - \frac{1}{2}e^{-\frac{1}{2}}$

Problem 2 :$y^1(x) = 80 - \frac{40.y(x)}{(2000-5x)}, y(0) = 100$

Exact solution:
$y(x) = 2(000 - 5x) - \frac{3900}{(2000)^9} \ (-2000 + 5t)$

Problem 3: $y^1(x) = 8(x - y(x)) + 1, y(0) = 2, h = 0.01$
Exact solution: $y(x) = x + 2e^{-8x}$

Table 1: Approximate Solution to Problem 1 with New Block Methods Derived&Adekunle/Adesanya

X	Exact Solution	Method A	Method B	Adekunle/Adesanya
0.1000	0.524385287749643	0.524385287750321	0.524385291472885	0.5243852877552174
0.2000	0.54758129098202	0.547581290982656	0.547581292362439	0.5475812909859664
0.3000	0.569646011787471	0.569646011788109	0.569646016843449	0.5696460117956543
0.4000	0.590634623461009	0.590634623462199	0.590634631475029	0.5906346234953703
0.5000	0.610599608464298	0.610599608465422	0.610599614227274	0.6105996086572718
0.6000	0.629590889659141	0.62959088966024	0.629590898362582	0.6295908898470451
0.7000	0.647655955140644	0.647655955142192	0.647655966177859	0.6476559553183269
0.8000	0.66483997698218	0.664839976983648	0.664839985880022	0.6648399771546479
0.9000	0.681185924189114	0.681185924190533	0.681185935425796	0.6811859243738679
1.0000	0.696734670143684	0.696734670145466	0.696734683206392	0.6967346704442603

Table 2: Absolute Error of Problem 1

X	Error of Method A	Error of Method B	Error of Adekunle&Adesanya
0.1000	6.78013E-13	3.72324E-09	5.574430E-012
0.2000	6.35936E-13	1.38042E-09	3.946177E-012
0.3000	6.38045E-13	5.05598E-09	8.183232E- 012
0.4000	1.18994E-12	8.01402E-09	3.436118E-011
0.5000	1.1241E-12	5.76298E-09	1.929743E-10
0.6000	1.09901E-12	8.70344E-09	1.879040E-10
0.7000	1.54798E-12	1.10372E-08	1.776835E-10
0.8000	1.46805E-12	8.89784E-09	1.724676E-10
0.9000	1.41909E-12	1.12367E-08	1.847545E -10
1.0000	1.78202E-12	1.30627E-08	3.005770E-10

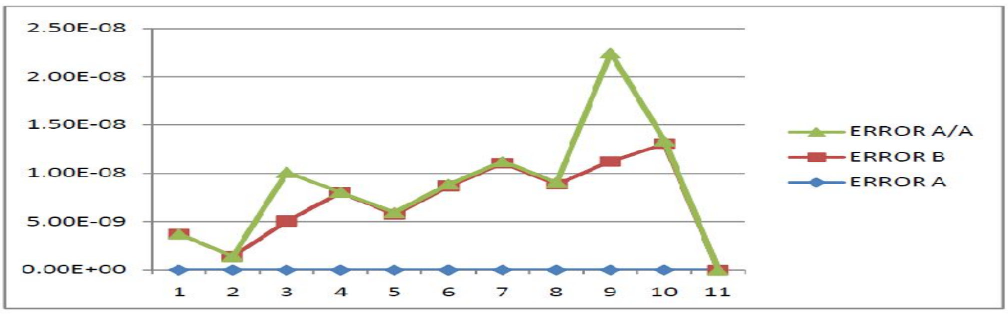


Table 3: Approximate Solution to Problem 2 with New Block Methods Derived

X	Exact Solution	Method A	Method B
0.1000	107.76623011683	107.76623011683	107.766230116832
0.2000	115.51494091931	115.5149409193	115.514940919303
0.3000	123.24616305089	123.246163050885	123.246163050887
0.4000	130.95992710909	130.959927109091	130.959927109095
0.5000	138.65626364555	138.656263645542	138.656263645546
0.6000	146.33520316601	146.335203166015	146.33520316602
0.7000	153.99677613051	153.996776130511	153.996776130518
0.8000	161.6410129533	161.641012953303	161.641012953309
0.9000	169.26794400299	169.267944002999	169.267944003007
1.0000	176.87759960259	176.877599602597	176.877599602605

Table 4: Absolute Error of Problem 2

X	Error of Method A	Error of Method B
0.1000	0	2E-12
0.2000	1E-11	7.01E-12
0.3000	5E-12	3E-12
0.4000	1.02E-12	5E-12
0.5000	8.01E-12	4.01E-12
0.6000	5E-12	1E-11
0.7000	9.95E-13	7.99E-12
0.8000	3.01E-12	9.01E-12
0.9000	9.01E-12	1.7E-11
1.0000	6.99E-12	1.5E-11

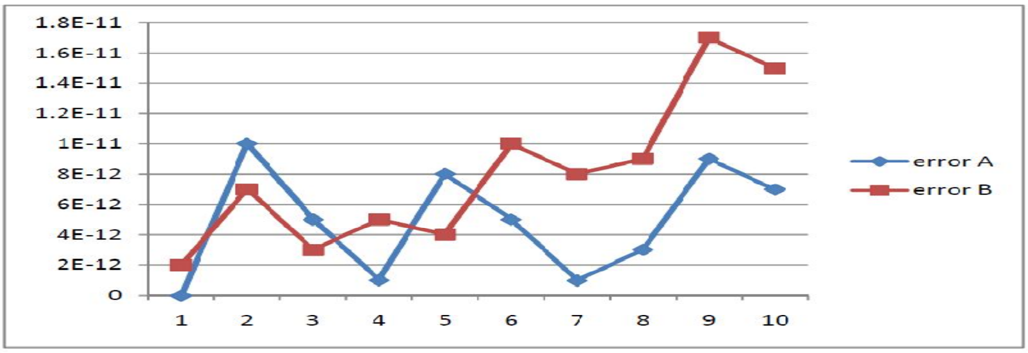
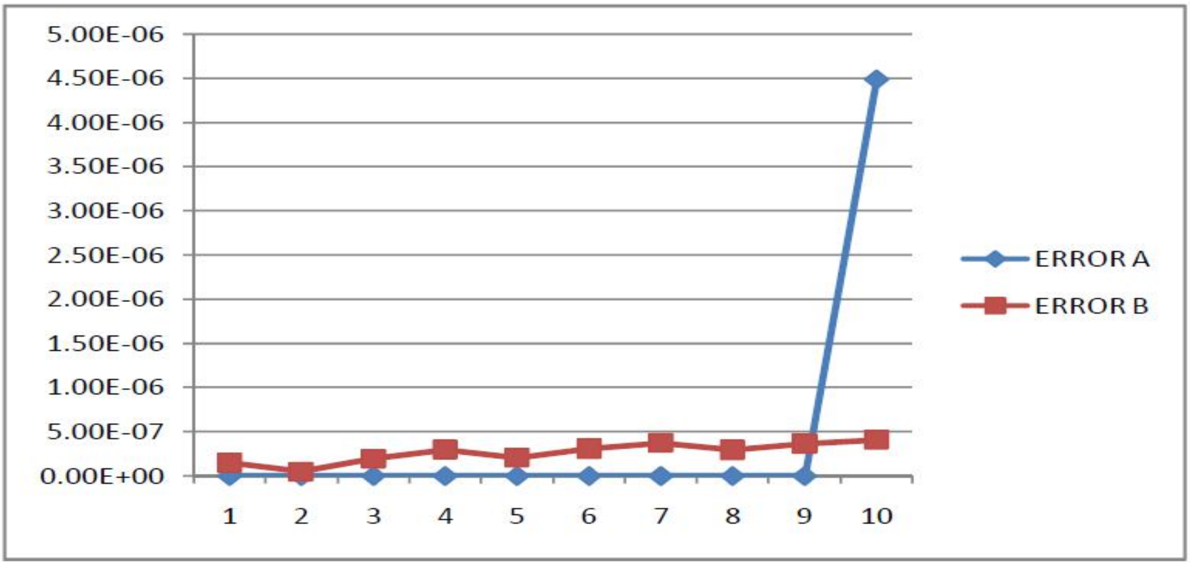


Table 5: Approximate Solution to Problem 3 with New Block Methods

X	Exact Solution	Method A	Method B
0.01	1.85623269277327	1.856232692705351.85623254580248	
0.02	1.72428757793242	1.72428757787052	1.85623254580248
0.03	1.60325572213311	1.60325572207262	1.60325552755727
o.o4	1.49229807414738	1.4922980740381	1.49229777891994
0.05	1.39064009207128	1.39064009197104	1.39063988675743
0.06	1.29756678361228	1.29756678351711	1.29756647749475
0.07	1.21241812769763	1.21241812756773	1.21241775417246
0.08	1.1345848480861	1.13458484796669	1.13458455615243
0.09	1.06350451191994	1.06350451180764	1.06350415071909
0.10	0.998657928234444	0.998653439493084	0.998657523265689

Table 4: Absolute Error of Problem 2

X	Error of Method A	Error of Method B
0.01	6.79199E-11	1.46971E-07
0.02	6.19E-11	5.02236E-08
0.03	6.04901E-11	1.94576E-07
0.04	1.0928E-10	2.95227E-07
0.05	1.0024E-10	2.05314E-07
0.06	9.51699E-11	3.06118E-07
0.07	1.299E-10	3.73525E-07
0.08	1.1941E-10	2.91934E-07
0.09	1.123E-10	3.61201E-07
0.10	4.48874E-06	4.04969E-07



DISCUSSION OF RESULT

In problem 1 table 1 the result using continues linear multi-step method (J. Sunday and Odekunle [8]) method and the two present method were compared and found out that the Adams type method exhibited a higher degree of accuracy, whereas in problem 2 table 2. Comparison was made only between the two present methods and Adam Type Method has a slight difference in degree of accuracy unlike in problem 3 table where the Adams Type Method performed best than the convectional method. This shows that Adams Type Method is better than conventional method.

CONCLUSION

We conclude that the Adams Type Method is of uniform order 7 and the other Adams Conventional Method is of uniform order 4 all at k = 3 are suitable for the solution of first order differential equation all are zero stable. For further suggestions, Adams Type Method can equally be compared with Adams-Bashforth Method, Backward Difference Method (BDF) and RungeKulta Type Method.

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