OPTIMIZATION OF THE LONGITUDINAL FINS WITH DIFFERENT GEOMETRIES FOR INCREASING THE HEAT TRANSFER

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Abstract - The main aim of this study is to obtain an optimum design point for fin geometry, so that heat transfer rate reaches to a maximum value in a constant fin volume. Effect of fin thicknesses ratio (τ), convection coefficient power index (m), profile power parameter (n), base thickness (δ) and fin material are evaluated in the fin optimization point for heat transfer rate, effectiveness and efficiency. It's assumed that the thickness of longitudinal fins varies with length in a special profile, so four different shapes (rectangular, convex, triangular and concave) are considered. In present study, temperature-dependent heat generation, convection and radiation are considered and an analytical technique based on the least square method is proposed for the solution methodology.

Keywords - Optimization; Longitudinal fins; Least Square Method (LSM); Heat generation; effectiveness.

I. INTRODUCTION

Fins or extended surfaces have many applications in heat transfer such as air conditioning, refrigeration, automobile, chemical processing equipment and electrical chips for increasing the heat transfer rate. Recently, researchers and engineers in this field have been paying more attention to both energy conservation and equipment size reduction due to energy crisis in the world. So, optimizing the heat transfer is highly desired for effective energy utilization. Because the weight and material cost of the extended surfaces are very important, fin dimensions should be optimized so that the least amount of fin material be used to dissipate a given amount of heat flow, or alternatively that the highest dissipation rate be obtained from a given volume of fin material. Also, using porous fins is another way for increasing the heat transfer in a constant volume of equipments which are discussed by Kiwan and Al-Nimr [1] and Kiwan [2] and many other researchers. Following some valuable researches in the fin optimizations are introduced.

Copiello and Fabbri [3] investigated the optimization of the heat transfer from wavy fins cooled by a laminar flow under conditions of forced convection and from a multi-objective point of view by finite element method (FEM) and genetic algorithm (GA) to find the geometries that maximize the heat transfer and minimize the hydraulic resistance at the same time. In another study, the optimum geometric dimensions of the parabolic profile circular fin with a constant volume, which yields the maximum heat dissipation, are determined by Xiang [4]. Sharqawy and Zubair [5] found optimized dimensions for maximize heat transfer in a fully wet longitudinal fins considering heat and mass transfer. Also they [6] optimized a uniform rectangular fin when the surface fin is dry, partially and fully wet.

In a different study, Kundu [7] considered a fin with a step reduction in cross section (SRC) and investigated the effect of various design and psychometric parameters on the fin performance of SRC fins and compared it is with the corresponding uniform cross section (UC) fin. Also, he and his co-workers [8] designed a T-shape fin with maximum heat transfer by obtaining the lengths and thicknesses. Furthermore, they [9] obtained performance and optimum design of porous fin with various profiles operating in convection environment. In another study, Zhang and Chung [10] determined the optimal dimensions of a radiating convecting annular fin using an arbitrary profile and more specifically to present convenient design charts for the thermal designers. They considered heat transfer rate, effectiveness and efficiency for optimum fin design. According to their study, we considered these parameters for our optimum design in current study.

Turkyilmazoglu [11] found exact solutions for thermal diffusion in a straight fin with varying exponential shape when the thermal conductivity and heat transfer coefficients are temperature dependent by power laws. He revealed that the efficiency and base heat transfer rate of the exponential profiles are higher than those of the rectangular fin. Aziz and Beers-Green [12] investigated the performance and optimum design of a longitudinal rectangular fin attached to a convectively heated wall of finite thickness by numerical method obtained by Maple package and they compared their results by those obtained by Adomian's decomposition and the differential quadrature method, also Khani and Aziz [13] applied Homotopy Analysis Method (HAM) for predicting the thermal performance of a straight fin of trapezoidal profile when both the thermal conductivity and the heat transfer coefficient are temperature dependent. Fin efficiencies for four different longitudinal solid fins such as rectangular, triangular, convex and exponential are considered by Torabi and Zhang [14] through Differential Transformation Method (DTM) considering all nonlinearities. Also Joneidi et al. [15] and Mosayebidorcheh and Mosayebidorcheh [16] used the same method to determine the fin efficiency of convective straight fin and Ganji et al. [17] used HPM analytical method for thermal analysis of annual fins. Another high accuracy analytical method which is used for solving the problem, is Least Square Method (LSM) which is widely used by authors [18-23] in solving different engineering problems such as fins thermal analysis.

Motivated by above mentioned works, in present study authors aim to use LSM for obtaining the temperature distribution in longitudinal fins with different section shapes and heat generation. Also, a complete optimization study for fin geometry and material is performed to reach an optimum point for the maximum heat transfer rate, effectiveness and fin efficiency.

II. MATHEMATICAL ANALYSIS

2.1. Governing equations

Consider an one-dimension longitudinal fins of rectangular, convex parabolic, triangular and concave parabolic profiles with the length *L* as shown in Fig. 1. The heat transfer from the fin surface is both convection and radiation. Suppose the effective sink temperature for radiative heat transfer is T_s and the temperature of surrounding fluid is T_a. The heat transfer of the fin tip is assumed negligible. The convective heat transfer *h*, thermal conductivity *k* and surface emissivity ε are considered to be functions of temperature of forms [14]

$$h = h_b\left(\frac{T-T_a}{T_b-T_a}\right)^m \tag{1}$$

$$\varepsilon = \varepsilon_s\left[1+\beta\left(T-T_s\right)\right] \tag{2}$$

where T_b is the base temperature of fin, h_b is the convection heat transfer at the temperature T_a, ε_s is the surface emissivity at the temperature of radiation sink T_s and m and β are constants. The constants m and β are measures of variation of convection heat transfer and surface emissivity with temperature, respectively. The power *m* in Eq. (1) depends on the heat transfer mode. Typical values of *m* are -1/4 for condensation or laminar film boiling, 1/4 for laminar natural convection, 1/3 for turbulent natural convection, 2 for nucleate boiling and 3 for radiation heat transfer [24, 25]. Also, it can be assumed that the radiation heat transfer between the fin and surrounding fluid (for example air) is negligible.

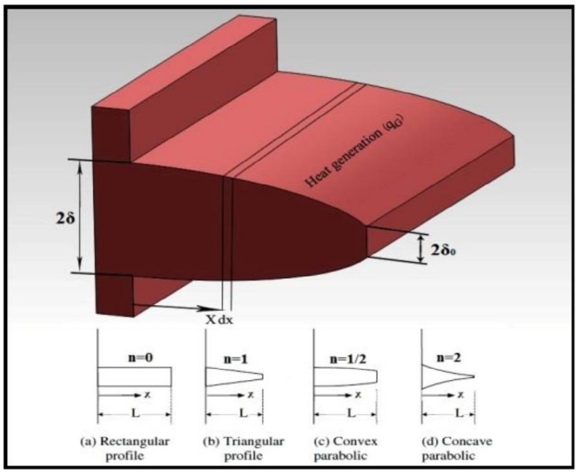

Fig 1. Schematic of the Longitudinal Fins

Some cases of the Murray-Gardner assumptions (1)-(2) are considered to obtain the balance energy equation of fin:

- Heat conduction in the fin is steady state.
- The fin material is isotropic and has constant properties.
- The temperature of the medium surrounding the fin is uniform.
- The base temperature of the fin is uniform.
- The heat transfer through the tip fin is small compare with the heat leaving its surface.

The heat transfer equation of the fin regarding the unit width is as follows:

$$2\frac{d}{dX}\left[kt\left(X\right)\frac{dT}{dX}\right]-h_b\left(\frac{T-T_a}{T_b-T_a}\right)^m\left(T-T_a\right) -\sigma\varepsilon_s\left[1+\beta\left(T-T_s\right)\right]\left(T^4-T_s^4\right)+2at\left(X\right)=0 \tag{3}$$

where k is the thermal conductivity, σ is the Stefan–Boltzmann constant and a is the internal heat generation of the fin. Regarding Fig. 1 the fin profile $t\left(X\right)$ can be formulated as:

$$t\left(X\right)=\left(\delta-\delta_0\right)\left(1-\frac{X}{L}\right)^n+\delta_0 \tag{4}$$

where δ and δ_0 are the local semi-fin thicknesses at the first and end, respectively. The value of n shows the type of fin profile. The profile number n is 0, 1/2, 1 and 2 for rectangular, convex, triangular and concave shapes, respectively which are shown in Fig. 1 schematically. The boundary conditions in the base and tip of fin give:

$$T\left(0\right)=T_b,\quad \left.\frac{dT}{dX}\right|_{X=L}=0 \tag{5}$$

Employing the following dimensionless parameters:

$$\theta=\frac{T}{T_b},\quad \theta_a=\frac{T_a}{T_b},\quad \theta_s=\frac{T_s}{T_b},\quad m_c=\frac{h_b L^2 T_b^{\,m}}{k\,\delta\left(T_b-T_a\right)^m}$$
$$\alpha=\beta T_b,\quad m_r=\frac{\sigma\varepsilon_s L^2 T_b^3}{k\,\delta},\quad \omega=\frac{aL^2}{k\,\delta},\quad \tau=\frac{\delta_0}{\delta} \tag{6}$$

The formulation of the balance energy reduces to:

$$2\frac{d}{dx}\left[\left((1-\tau)(1-x)^n+\tau\right)\frac{d\theta}{dx}\right]-m_c(\theta-\theta_a)^{m+1}$$
$$-m_r\left(1+\alpha(\theta-\theta_s)\right)\left(\theta^4-\theta_s^4\right)+2\left((1-x)^n+\tau\right)\omega=0 \quad (7)$$

with the following dimensionless boundary conditions:

$$\theta(0)=1, \qquad \left.\frac{d\theta}{dx}\right|_{x=1}=0 \quad (8)$$

The non-dimensionless parameters such as the profile number n, the thickness ratio τ, the radiation characteristic number m_r and the convection characteristic number m_c play important roles temperature distribution and heat dissipation of the fin.

2.2. Heat dissipation

This work is organized for the constant fin volume. Regarding this, to determine optimal dimension of longitudinal fin, we maximize the value of the heat transfer for a constant fin volume which is given by [10]

$$V=\int_{X=0}^{L}2t(X)dX=2\delta L\left(\frac{1-\tau}{n+1}+\tau\right) \quad (9)$$

The heat dissipation from the fin surface is equal to heat transfer at the fin base.

$$q=-2k\delta\left.\frac{dT}{dX}\right|_{X=0} \quad (10)$$

Combination Eqs.**Error! Reference source not found.** and **Error! Reference source not found.**, we have:

$$q=\frac{-kT_bV}{L^2\left(\frac{1-\tau}{n+1}+\tau\right)}\left.\frac{d\theta}{dx}\right|_{x=0} \quad (11)$$

Here, we define the dimensionless heat transfer Q in the following form:

$$Q=\frac{qL^2}{kT_bV}=\frac{-1}{\left(\frac{1-\tau}{n+1}+\tau\right)}\left.\frac{d\theta}{dx}\right|_{x=0} \quad (12$$

2.3. Fin efficiency

Fin efficiency is defined the ratio of actual heat transfer from the fin to ideal heat transfer of a fin of same geometry and conditions but with an infinite conductivity (the temperature of the entire fin surface is equal to the fin base temperature). So, the ideal heat transfer can be obtained from

$$q_{ideal}=2Lh_bT_b(1-\theta_a)+2L\sigma\varepsilon_sT_b^4\left(1+\alpha(1-\theta_s)\right)(1-\theta \quad (13)$$

The fin efficiency may be found as [14]

$$\eta=\frac{q}{q_{ideal}}=\frac{-1}{\left(\frac{h_bL}{k}\right)\frac{L}{\delta}(1-\theta_a)+m_r\left(1+\alpha(1-\theta_s)\right)(1-\theta_s^4)}\left.\frac{d\theta}{dx}\right|_{x=0} \quad (14)$$

2.4. Fin effectiveness

The fin effectiveness is one of the important variables in heat transfer analysis of the fins. In the present work, we use the Gardner definition of fin effectiveness. It is the ratio of the actual heat dissipation of fin to the heat transfer if the fin wasn't existing (2)-(3). In other word, the fin effectiveness expresses how much extra heat is being transferred by the fin. The heat dissipation without fin can be obtained from:

$$q_p=2\delta h_bT_b(1-\theta_a)+2\delta\sigma\varepsilon_sT_b^4\left(1+\alpha(1-\theta_s)\right)\left(1-\theta_s^4\right) \quad (15)$$

Thus, the following expression will be adopted for fin effectiveness [10]

$$\zeta=\frac{q_{actual}}{q_p}=\frac{-1}{\left(\frac{h_bL}{k}\right)(1-\theta_a)+m_r\frac{\delta}{L}\left(1+\alpha(1-\theta_s)\right)\left(1-\theta_s^4\right)}\left.\frac{d\theta}{dx}\right|_{x=0} \quad (16)$$

The value of ζ must be greater than unity for employment of the fin to be worthwhile. In other word, economic considerations may dictate the fin effectiveness must exceed unity. The fin act as an insulator for the case $\zeta<1$.

2.5. Generation number

The generation number is the ratio of the internal heat generation to the ideal heat dissipation of the fin if the entire fin were to operate at the base temperature. The value of the internal heat generation may be computed from [14]

$$q_G=\int_{X=0}^{L}2a\delta\left(\left(1-\frac{X}{L}\right)^n+\tau\right)dX \quad (17)$$

Thus, we have for generation number

$$N_G=\frac{q_G}{q_{Ideal}}=\frac{\omega\left(\frac{1-\tau}{n+1}+\tau\right)}{\left(\frac{h_bL}{k}\right)\frac{L}{\delta}(1-\theta_a)+m_r\left(1+\alpha(1-\theta_s)\right)\left(1-\theta_s^4\right)} \quad (18)$$

III. RESULTS AND DISCUSSION

Results are depicted in Fig. 2-a for different convection coefficient power index (m) and Fig. 2-b for different fin materials which their physical properties are presented in Table 1. As seen in this figures, copper has higher fin tip temperature due to higher thermal conductivity. Also, concave section has the highest and rectangular section has the lowest temperature distribution in whole fin length. Furthermore it can be concluded that when convection coefficient is temperature-dependent (m=2), fin temperatures are more than independent cases because of lower heat transfer rate to ambient. Comparison of the results of LSM with numerical method which confirms the accuracy of this analytical method as Hatami et al. [18-23] mentioned. In this section we want to optimize the fin design to reach a maximum heat transfer rate in constant volume. For this aim, a computer program is written for the optimization problems not only to obtain the optimum geometric dimensions of the longitudinal fin of different profiles, but also to study the effects of the three physical parameters *n,m* and δ on the optimum geometric dimensions. Furthermore, the effect of three materials on its performance is considered. Constructions of the following figures (Figs. 3-8) for optimization analysis are as follow: a) Heat transfer rate according to Eq. (12), b) Fin effectiveness according to Eq. (16) and c) Fin efficiency according to Eq. (14). In these figures constants parameters considered as,

$$T_b = 400, T_a = 300, T_s = 300,\ h_b = 50,\ \beta = ($$

and each figures show the effect of some physical parameters on a) Heat transfer, b) effectiveness and c) efficiency which are discussed completely in the following.

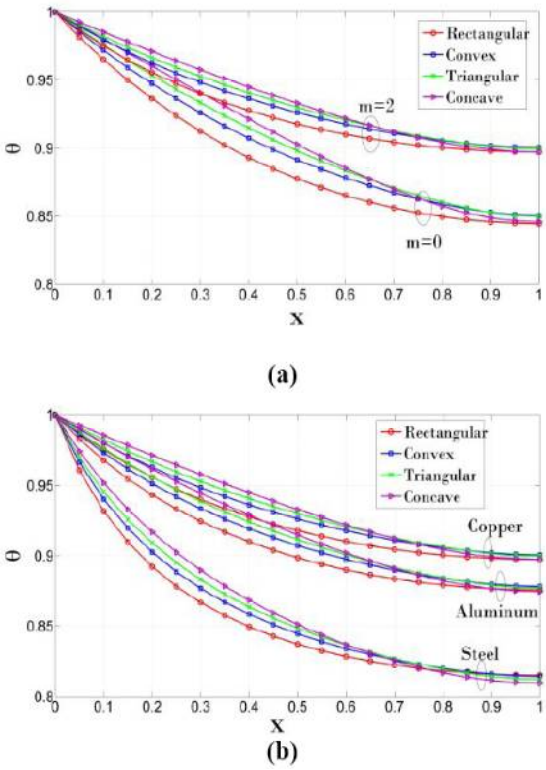

Fig. 2 Temperature distribution along the fins obtained by LSM and Num. a) for copper in different section shapes and *m* number b) for different material and section shapes where τ=1/5, *m*=2 and V=0.05

An optimization for fin geometry is performed in Fig. 3 when fin volume is constant. As seen by increasing the profile power index (*n* in Eq. (4)), the rate of heat transfer (and consequently efficiency) initially increases, then reaches to a maximum value and finally starts declining. The peak of the curves indicates an optimum design condition for a specified fin volume. Also, effect of thicknesses ratio (τ) on optimum point is shown in these figures. As shown, by increasing the τ, optimum point occurred in lower *n* values and effectiveness increases.

Fig. 4 demonstrates the effect of convection coefficient power index (*m*) on optimization point. As described in section 2-1, the *m* values are depended on heat transfer mode and so these values are considered.

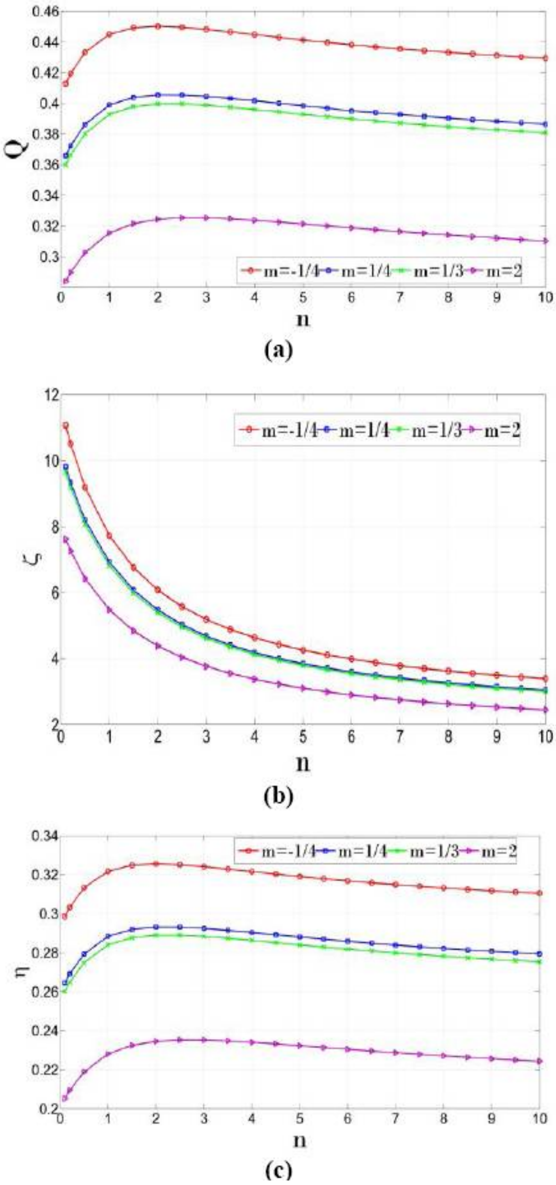

Fig. 4 Effect of n and m parameters on a) non-dimensional heat transfer b) effectiveness and c) efficiency for copper fin

By increasing the *m* parameter, all of the performance parameters are decreased and no significant effect on optimum point of geometry (*m*) is observed. To investigate the influence of fin material on heat dissipation rate, Fig. 5 is depicted. It’s obvious that copper has the most heat transfer rate among the other materials. The same holds true for the effects of the material on both of the effectiveness and fin efficiency in constant fin volume.

Table 1. Thermo physical properties of porous material

Material full name	ρ (kg m-3)	ks (W K-1 m-1)	Cp (kJ kg-1 K-1)
Copper	8960	401	0.386
Aluminum	2700	235	0.897
Steel	7750	41	0.491

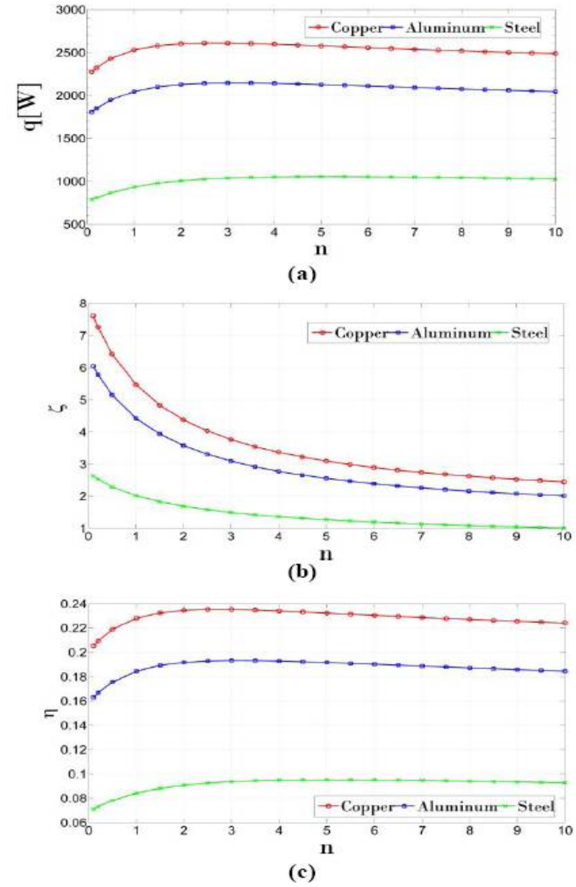

Fig. 5 Effect of n parameter and different materials on a) non-dimensional heat transfer b) effectiveness and c) efficiency

CONCLUSIONS

Many of investigation are still constructed to find out the optimum fin geometry on the basis of enhancement of heat transfer rate as well as ease of fabrication. The present study works on the performance and optimum design analysis of straight convective-radiative fins with internal heat generation of four special different geometries, namely, rectangular, convex, triangular and concave types. The following main points can be concluded from the present study. By increasing the convection coefficient power index (*m*), optimum points for profile power index of fin geometry (n) increases as well as optimum point for fin thicknesses (δ).By increasing the fin thicknesses ratio (τ), maximum heat transfer rate decreases and occurred in lower *n* values.

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