HIGH ORDER SLIDING MODES FOR VEHICLE SUSPENSIONS VIA OPTIMIZED SUPER TWISTING ALGORITHM

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Abstract: A high order sliding mode controller, based on super twisting algorithm, is designed in this study for vehicle active suspensions. High order sliding mode is preferred since the first order sliding mode controller suffer from chattering in control signal. The controller parameters are determined using a multi-objective genetic algorithm. Designed controller is applied to a quarter car active suspension system, where it is aimed to improve the ride comfort of passengers. First order sliding mode controller is also applied for comparison. Numerical results indicate the outstanding performance of the designed high order sliding mode controller.

Keywords: High Order Sliding Mode Controller, Super Twisting Algorithm, Multi-Objective Genetic Algorithm, Quarter Vehicle Model.

I. INTRODUCTION

Passive vehicle suspensions are composed of spring and dampers placed parallel to each other between vehicle body and wheel-axle assembly. For normal operation conditions their performance is fixed. To improve ride comfort of passengers, semi-active suspensions equipped with electrorheological or magnetorheological dampers may be used. Further improvement is also possible by using active suspensions that include actuators providing external energy to the system. Extensive studies concerning active suspensions have been carried out during last decades and various control methods, have been proposed such as fuzzy logic control [1-2], H∞ control [3], and backstepping control [4]. As a variable structure controller [5], sliding mode controller (SMC) can be applied to non-linear and uncertain systems [6-7]. When the system is on the sliding surface the closed-loop control system is insensitive to external disturbances and parameter variations [6]. Active suspension control has been one of the main application areas of the SMC method. Yagiz et al. [8] designed a fuzzy SMC and applied it to a nonlinear half car model. Deshpande et al. [9] investigated active control of the vibrations of a vehicle model using sliding mode control with disturbance observer (DOSMC). Alves et al. [10] have proposed a sliding mode controller for active suspension system with data acquisition delay. They have used two approaches including continuous-time and a discrete-time control. On the other hand, in sliding mode controlled systems high frequency oscillations may arise in control signal and states of the system that is called as chattering. This may harm the mechanical components of the system, such as the actuators. Several methods have been proposed to overcome this phenomenon, where continuous approximations for the signum function such as saturation [11] and sigmoid-like functions [12] are used. On the other hand the resulting motion is not ideal sliding motion. Therefore, high order SMC method has been proposed in literature to attenuate or almost prevent chattering [13-14]. Especially, the Super-Twisting Algorithm (STA) which is a well-known second order sliding mode (SOSM) algorithm is introduced by Levant [14], and it has been used for control [14-15-16].

A high order sliding mode controller (HOSMC) based on super twisting algorithm is designed for a quarter vehicle active suspension system in this study. Additionally, parameters of the designed controller are found using a multi-objective genetic algorithm. In order to verify the performance and chattering attenuation of the designed controller, a first order sliding mode controller is also designed and applied to the quarter vehicle model for comparison. Designed controllers are tested via simulations on a quarter car vehicle suspension model.

II. CONTROLLER DESIGN

2.1. First Order Sliding Mode Controller Design

In sliding mode controlled systems, the system is insensitive to parameter changes and external disturbances on the sliding surface and this property attracts researchers very much. The state equations of a dynamic system may be written as follows:

$$\dot{x}_1 = x_2 \quad (1)$$

$$\dot{x}_2 = f(x_1, x_2) + g(x_1, x_2)\bar{u} + \bar{d} \qquad (2)$$

Here x_1 and x_2 are states, $\bar{u}$ is the control signal and $\bar{d}$ is a bounded external disturbance ($\left|\bar{d}\right| \le \Delta$, $\Delta > 0$). Sliding surface is chosen as:

$$\sigma = \alpha\left(x_{1r} - x_1\right) + \left(\dot{x}_{1r} - \dot{x}_1\right) \qquad (3)$$

where $\alpha > 0$ for stability and subscript r stand for reference value. The following Lyapunov function candidate has to be positive definite and its derivative has to be at least negative semi-definite.

$$V = \frac{1}{2}\sigma^2 \qquad (4)$$

$$\dot{V} = \sigma\dot{\sigma} \quad (5)$$

If the limit condition is applied to (5) then

$$\dot{\sigma} = \alpha\left(\dot{x}_{1r} - \dot{x}_1\right) + \left(\ddot{x}_{1r} - \ddot{x}_1\right) = 0 \qquad (6)$$

Using (1)-(2)and (6), the control law for the limit case that is, equivalent control $\bar{u}_{eq}$ is obtained as below:

$$\bar{u}_{eq} = g^{-1}(x_1, x_2)\left\{\alpha\left(x_{2r} - x_2\right) + \dot{x}_{2r} - f(x_1, x_2)\right\} \qquad (7)$$

Let the total control signal is chosen to be;

$$\bar{u} = \bar{u}_{eq} + k\, g^{-1}(x_1, x_2)\,\mathrm{sign}(\sigma) \qquad (8)$$

Then the derivative of the Lyapunov function becomes:

$$\begin{aligned}\dot{V} &= \sigma\dot{\sigma} \\ &= \sigma\left\{\alpha\left(x_{2r} - x_2\right) + \left(\dot{x}_{2r} - f(x_1, x_2) - g(x_1, x_2)\bar{u} - \bar{d}\right)\right\} \\ &= -k\,\sigma\, sign(\sigma) - \sigma\bar{d} \\ &\le -\left|\sigma\right|\left(k - \Delta\right)\end{aligned} \qquad (9)$$

If k is chosen to be $k > \Delta$ then derivative of the Lyapunov function is negative definite $\dot{V} < 0$ and the system is forced to reach the sliding surface.

2.2. High Order Sliding Mode Controller (HOSMC) with Equivalent Control

A second-order sliding mode controller based on super twisting algorithm is presented in this section, that will include the equivalent control. According to Levant [14-15] the second-order sliding mode may be defined as the motion on non-empty set $\sigma = \dot{\sigma} = 0$ that consist of locally Flippov trajectories and where are continuous functions of the closed-system state variables. Now suppose that the system is defined as:

$$\dot{x}_1 = x_2 \quad (10)$$

$$\dot{x}_2 = f(x_1, x_2) + g(x_1, x_2)\bar{u} + \bar{d} \qquad (11)$$

where x_1 and x_2 are states, $\bar{u}$ is the control signal and $\bar{d}$ is a bounded external disturbance. The sliding surface is selected as

$$\sigma = \alpha\left(x_{1r} - x_1\right) + \left(\dot{x}_{1r} - \dot{x}_1\right) \qquad (12)$$

where $\alpha > 0$. By taking the time derivative of the sliding surface and using (10)and (11).

$$\dot{\sigma} = \alpha\left(x_{2r} - x_2\right) + \dot{x}_{2r} - f(x_1, x_2) - \mathrm{g}(x_1, x_2)\bar{\mathrm{u}} - \bar{d} \qquad (13)$$

Then by defining following variables

$$\phi(x) = \alpha\left(x_{2r} - x_2\right) + \dot{x}_{2r} - f(x_1, x_2) \quad (14)$$

$$u = -g(x_1, x_2)\bar{u} \quad (15)$$

$$d = -\bar{d} \quad (16)$$

the derivative of sliding surface function can be rewritten as

$$\dot{\sigma} = \phi(x_1, x_2) + \mathrm{u} + d \qquad (17)$$

Here it is assumed that disturbance d is bounded as $\left|d\right| \le \Delta\left|\sigma\right|^{1/2}$, $\Delta > 0$. For the limit case $\dot{\sigma} = 0$ and nominal system that is $d = 0$, the equivalent control u_{eq} is found as:

$$\mathrm{u}_{eq} = -\phi(x_1, x_2) \quad (18)$$

For the discontinuous part u_{dc} of the total control law we use the super twisting algorithm presented in [14].

$$\mathrm{u}_{dc} = -k_1\left|\sigma\right|^{1/2} sign(\sigma) + v \qquad (19)$$

$$\dot{v} = -k_2 sign(\sigma) \quad (20)$$

Therefore the total control signal is,

$$u = u_{eq} + u_{dc} \qquad (21)$$

For the stability analysis of the closed-loop system, following Lyapunov function, proposed in [17], is used.

$$V = 2k_2\left|\sigma\right| + \frac{1}{2}\upsilon^2 + \frac{1}{2}\left(k_1\left|\sigma\right|^{1/2} sign(\sigma) - \upsilon\right)^2 \quad (22)$$

This function can be arranged in a quadratic form as

$$V = \boldsymbol{\xi}^T \mathbf{P}\, \boldsymbol{\xi} \qquad (23)$$

$$\boldsymbol{\xi}^T = \left[\left|\sigma\right|^{1/2} sign(\sigma) \quad \upsilon\right] \quad (24)$$

$$\mathbf{P} = \begin{bmatrix} 2k_2 + \dfrac{k_1^2}{2} & \dfrac{-k_1}{2} \\ \dfrac{-k_1}{2} & 1 \end{bmatrix} \qquad (25)$$

The derivative of Lyapunov function can be calculated as

$$\begin{aligned}\dot{V} &= \dot{\boldsymbol{\xi}}^T \mathbf{P}\boldsymbol{\xi} + \boldsymbol{\xi}^T \mathbf{P}\dot{\boldsymbol{\xi}} \\ &= \dot{\sigma} sign(\sigma)\left(2k_2 + \frac{1}{2}k_1^2\right) - k_1\dot{\upsilon}\left|\sigma\right|^{1/2} sign(\sigma) - \frac{k_1\upsilon\dot{\sigma}}{2\left|\sigma\right|^{1/2}} + 2\dot{\upsilon}\upsilon\end{aligned} \qquad (26)$$

$$\dot{V} \le -\left(k_1 k_2 + \frac{k_1^3}{2}\right)\left|\sigma\right|^{1/2} + k_1^2\upsilon sign(\sigma) - \frac{k_1\upsilon^2}{2\left|\sigma\right|^{1/2}} + \Delta\left(2k_2 + \frac{1}{2}k_1^2\right)\left|\sigma\right|^{1/2} - \frac{k_1\upsilon}{2}\Delta sign(\sigma) \qquad (27)$$

This can be also arranged in a quadratic form as

$$\dot{V} \le -\frac{k_1}{2\left|\sigma\right|^{1/2}}\boldsymbol{\xi}^T \mathbf{Q}\,\boldsymbol{\xi} \qquad (28)$$

$$\mathbf{Q} = \begin{bmatrix} 2k_2 + k_1^2 - \left(\dfrac{4k_2}{k_1} + k_1\right)\Delta & -k_1 + \dfrac{\Delta}{2} \\ -k_1 + \dfrac{\Delta}{2} & 1 \end{bmatrix} \qquad (29)$$

If $k_1 > 2\Delta$, $k_2 > \dfrac{k_1\Delta^2}{8\left(k_1 - 2\Delta\right)}$ then the derivative of the Lyapunov function becomes negative definite $\dot{V} < 0$, and reaching to the sliding surface is guaranteed.

III. VEHICLE MODEL AND SIMULATION RESULTS

3.1. Quarter Vehicle Model

In this study a quarter car active suspension system is used to investigate the ride comfort improvement performance of the designed controllers. The quarter car suspension system is widely preferred by researchers since it has a simple structure and well represents the behavior of a real active suspension system. In the physical model shown in **Fig. 1.** the m_s (2.45kg) and m_{us} (1kg) stand for the sprung and unsprung masses; k_t (2500 N/ m) and k_s $(900\,N/m)$ are stiffness values for tire and suspension spring; b_t $(5\,Ns/m)$ and b_s $(7.5\,Ns/m)$ are the damping coefficients of the tire and suspension damper, respectively. Here y_0 is the road surface input to the system, y_s and y_{us} are vertical displacement of the sprung and unsprung masses and $\bar{u}$ stand for the actuator control signal. The mathematical model of the quarter vehicle active suspension system was obtained using Lagrange's equations. By choosing the state variables as

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} y_s & \dot{y}_s & y_{us} & \dot{y}_{us} \end{bmatrix}^T \quad (30)$$

The equations of the motion can be written in a convenient form for the controller design as

$$\dot{x}_1 = x_2 \quad (31)$$

$$\dot{x}_2 = -\frac{1}{m_s}\left[b_s\left(x_2 - x_4\right) + k_s\left(x_1 - x_3\right)\right] + \frac{1}{m_s}\bar{u} + \bar{d} \quad (32)$$

$$\dot{x}_3 = x_4 \quad (33)$$

$$\dot{x}_4 = \frac{1}{m_{us}}\begin{bmatrix} b_s\left(x_2 - x_4\right) - b_t\left(x_4\right) - k_t\left(x_3\right) \\ +k_s\left(x_1 - x_3\right) + b_t\dot{y}_0 + k_t y_0 \end{bmatrix} - \frac{1}{m_{us}}\bar{u} - \bar{d} \quad (34)$$

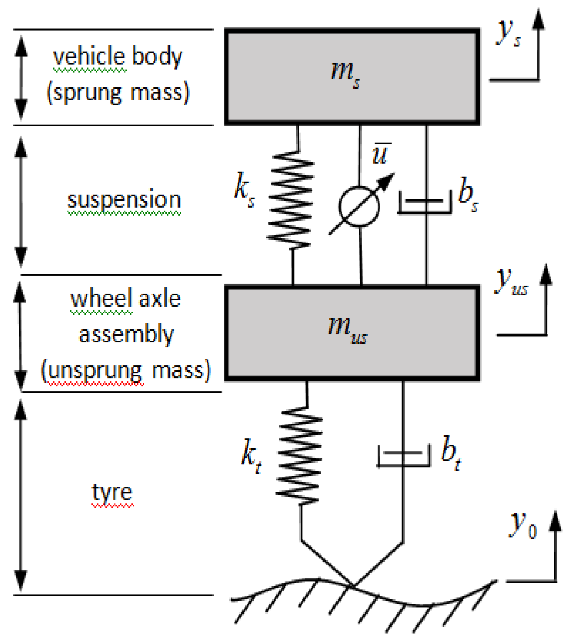

Fig.1. The quarter car model with active suspension.

Then by using the governing equations for the sprung mass, the control signal of the designed HOSMC for active suspension system is obtained as

$$\bar{u} = -m_s\,u \quad (35)$$

$$\begin{aligned} u = &-\left\{\alpha\left(x_{2r} - x_2\right) + \dot{x}_{2r} + \frac{b_s}{m_s}\left(x_2 - x_4\right) + \frac{k_s}{m_s}\left(x_1 - x_3\right)\right\} \\ &-k_1\left|\alpha\left(x_{1r} - x_1\right) + \dot{x}_{1r} - x_2\right|^{1/2} sign\left(\alpha\left(x_{1r} - x_1\right) + \dot{x}_{1r} - x_2\right) \\ &+\upsilon \end{aligned} \quad (36)$$

$$\dot{\upsilon} = -k_2\left\{sign\left(\alpha\left(x_{1r} - x_1\right) + \dot{x}_{1r} - x_2\right)\right\} \quad (37)$$

3.2. Choosing Optimum Controller Parameters with Multi-Objective Genetic Algorithm

By minimizing certain fitness functions, optimal parameters for the designed controller will be searched in this section. Genetic Algorithms (GAs) use principles inspired by genetic processes occurring in nature to obtain solutions to problems and they usually consist of reproduction, crossover and mutation operators. A fitness function must be devised for each problem to be solved [18]. The aim of Multi-Objective Optimization with Genetic Algorithm (MOGA) is minimization of multiple fitness functions simultaneously. It is used to solve multi objective optimization problems by identifying the Pareto front - the set of evenly distributed non dominated optimal solutions [19-20]. The optimization method used in this study can efficiently choose the appropriate gain parameters for the controllers based on several proposed fitness functions $\psi_i\left(i = 1,\ldots,5\right)$ given below. It is aimed to reduce accelerations of the vehicle body and attenuate possible chattering.

$$\psi_1 = \sum_{i=1}^{n}\left|y_{sri} - y_{si}\right| \quad (38)$$

$$\psi_2 = \frac{1}{\sqrt{n}}\left[\sum_{i=1}^{n}\left(\dot{u}_i\right)^2\right]^{1/2} \quad (39)$$

$$\psi_3 = \dim ension\left[\ddot{y}_{si} \rightarrow \begin{Bmatrix} \ddot{y}_{si} < 0 & and & \ddot{y}_{si-1} > 0 \\ \ddot{y}_{si} > 0 & and & \ddot{y}_{si-1} < 0 \end{Bmatrix}\right] \quad (40)$$

$$\psi_4 = \frac{1}{n}\sum_{i=1}^{n}\left|u_i - \frac{1}{n}\sum_{i=1}^{n}\left|u_i\right|\right| \quad (41)$$

$$\psi_5 = \frac{1}{n}\sum_{i=1}^{n}\left|\ddot{y}_{si} - \frac{1}{n}\sum_{i=1}^{n}\left|\ddot{y}_{si}\right|\right| \quad (42)$$

In this study the first fitness function ψ_1 stand for reference tracking performance of the controller and second fitness function ψ_2 stand for fluctuations in control signal. The third one ψ_3 denotes the number of crossing from zero for the acceleration signal. Fourth ψ_4 and fifth ψ_5 fitness functions denote the mean values for the magnitudes of the control signal and sprung mass acceleration. The flowchart of the control algorithm is shown in **Fig. 2.** The optimum value of gain parameters obtained by MOGA are presented in **Table 1** and they are used for the control of vehicle active suspension model.

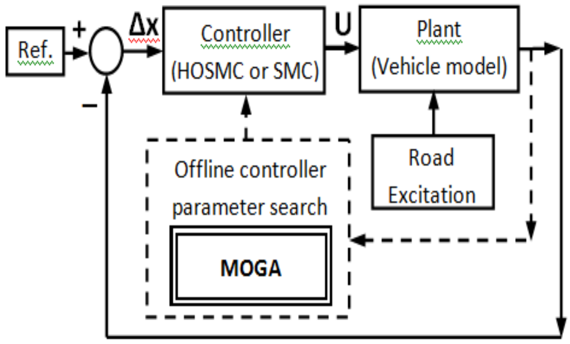

Fig.2.Block diagram for the controller

Table1: Optimum Controller Parameters Found Using MOGA

Controller	*Parameter*	*Value*
SMC	k	***0.55***
HOSMC	k_1	***1.43***
	k_2	***9.69***

3.3. Performance indicators

In this study several performance indicators defined by following equations will be used through evaluation of the simulation results.

$$RCI = \frac{1}{\sqrt{n}}\left[\sum_{i=1}^{n}\left(\ddot{y}_{si}\right)^2\right]^{1/2} \tag{43}$$

$$RHI = \frac{1}{\sqrt{n}}\left[\sum_{i=1}^{n}\left(k_t\left(y_{usi} - y_{0i}\right)\right)^2\right]^{1/2} \tag{44}$$

$$CEI = \frac{1}{\sqrt{n}}\left[\sum_{i=1}^{n}\left(u_i\right)^2\right]^{1/2} \tag{45}$$

$$CI = \frac{1}{\sqrt{n}}\left[\sum_{i=1}^{n}\left(\dot{u}_i\right)^2\right]^{1/2} \tag{46}$$

The ride comfort indicator (RCI) presents the root mean square (RMS) value of the sprung mass acceleration that reflects the acceleration perceived by the passengers. It is often used to quantify ride comfort. The road holding indicator (RHI) reflects the road holding of the vehicle. The control effort indicator (CEI) stand for the control effort used by the relevant controller and chattering indicator (CI) quantifies the chattering in the control signal.

IV. SIMULATION RESULTS WITH DISTURBANCE

Fig. 3 (a) shows the road input applied to the vehicle model. To investigate the performances of the designed controllers in the case of external disturbances, a sinusoidal disturbance with 1 Hz frequency and 1 N amplitude given in **Fig. 3 (b)** is also added to the control signal.

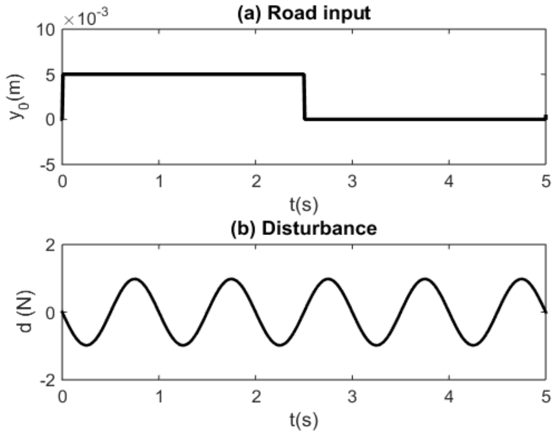

Fig.3.Road input and disturbance.

Simulation results with the applied road input and disturbance force are presented in **Fig. 4-5-6-7-8** along with performance indicators in **Fig. 9.** From those figures it is observed that the best improvement in ride comfort is obtained with designed HOSMC, since magnitudes for acceleration in **Fig. 5** and corresponding ride comfort indicator in **Fig. 9** is much more reduced if compared to the SMC case. It is also seen that chattering is reduced by using designed HOSMC if compared to the SMC in **Fig. 8**. From chattering indicator values in **Fig. 9** it is seen that HOSMC reduces chattering by approximately % 64 if compared with first order SMC.

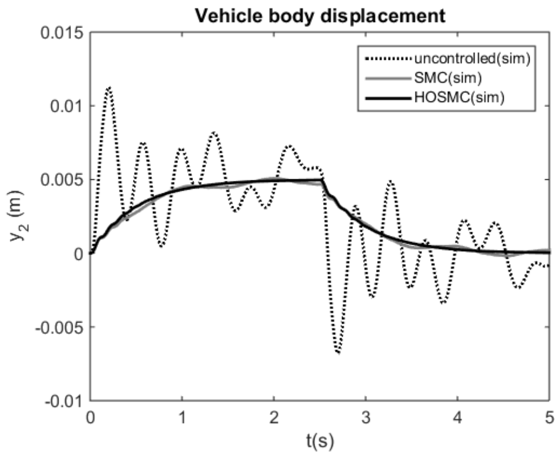

Fig.4.Vehicle body displacement.

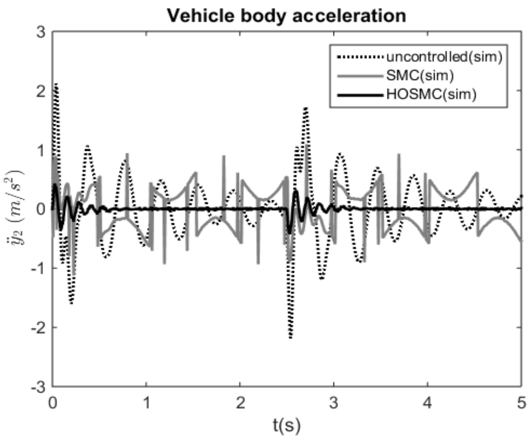

Fig.5.Vehicle body acceleration.

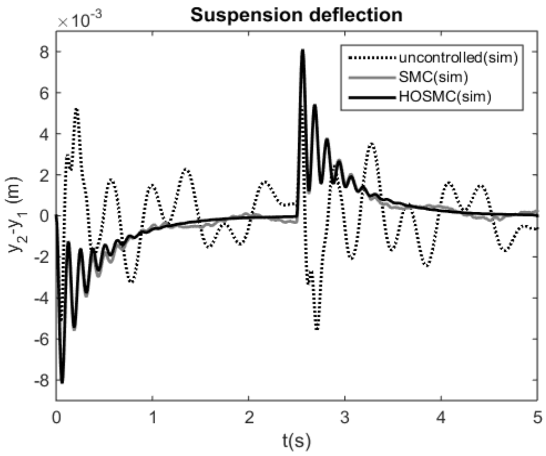

Fig.6.Suspension deflection.

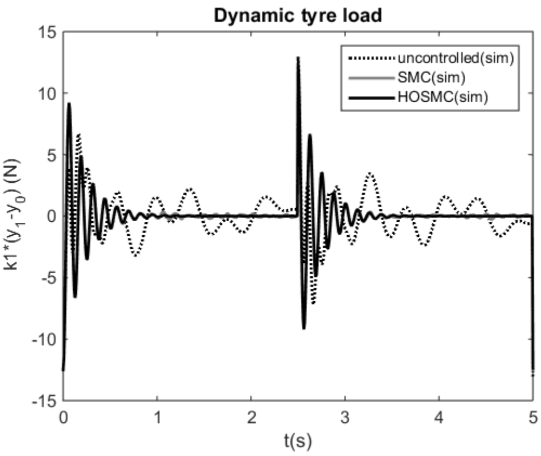

Fig.7.Dynamic tyre load.

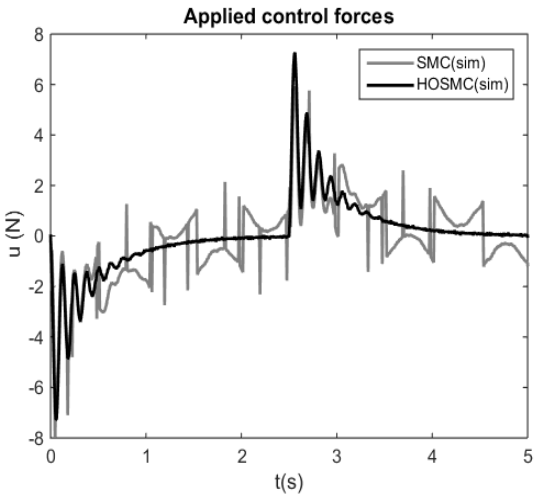

Fig.8.Applied control forces.

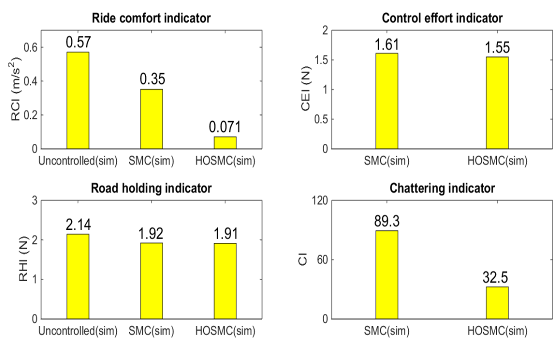

Fig.9.Performance indicators

CONCLUSIONS

A high order sliding mode controller has been designed for the quarter car active suspension system in this study. A first order sliding mode controller has also been designed for comparison purpose. The aim was to improve the ride comfort of passengers without decreasing road holding of the vehicle and especially to attenuate the possible chattering in control signal if compared with the first order sliding mode controller. The parameters of the designed controllers have been obtained by using multi-objective genetic algorithm searches. Extensive time responses have been obtained by simulations. It was deduced by investigation of those results that best ride comfort improvement has been achieved by the high order sliding mode controller. Furthermore, it is seen that high order sliding mode controller used less control effort with less chattering than the first order sliding mode controller, and this is preferable since high chattering may harm the actuator and other mechanical components of the system.

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